MT EDUCARE LTD.

ICSE X SUBJECT : **MATHEMATICS**

Algebra

STEP UP ANSWERSHEET

2003

1. $x = 2 \pm \sqrt{3}$

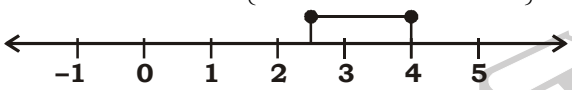
3. a = –2

4. $x = 6$

5. (i) 30
   (ii) 1200 seats

6. $x = 3$ and $y = 2$

7. Solution set is $\left\{x : 2\frac{1}{2} \le x \le 4, x \in \mathrm{R}\right\}$



2004

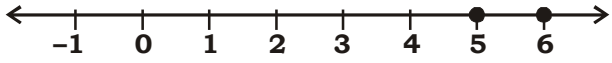
1. $\begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$

2. $(x + 1)\ (x - 2)\ (x + 2)$

3. $x = 6$

4. $x = 1.70, -1.37$

5. S.S. = {5, 6}



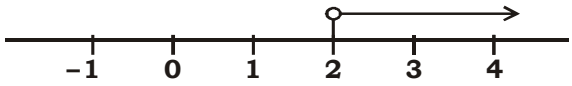
6. $x = 6$

2005

1. a = –3 and b = –1

2. 3

3. A ą B = {$x : x > 2, x$ Č R}



4. 6.53, –1.53

5. $x = 36$

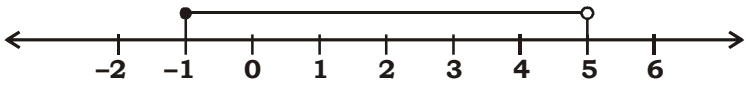
6. 30 km/h

2006

1. 25:6

2. $(2x + 7)\ (x - 2)\ (x + 1)$

3. {$x : -1 \leq x < 5, x \in$ R}



4. $\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

5. $x = 3.64, -0.14$

6. ₹ 45

2007

2. $y = -8$ and $x = 2$

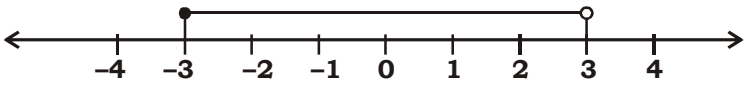
3. $\begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$

4. $(x - 1)\ (x - 2)\ (x - 4)$

5. $x = 4.85, -1.85$

6. (i) 5 years
   (ii) 30 years

7. {$x : -3 \leq x < 3, x \in$ R}

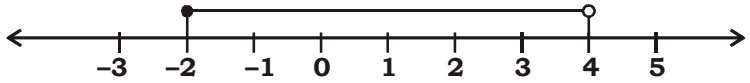


2008

1. 24

2. (i) p = 5
   (ii) $(x - 2)\,(2x + 1)\,(x + 1)$

3. $\{y : -2 \text{ á } y < 4\}$



4. $M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$

6. $x = 0.27, -2.27$

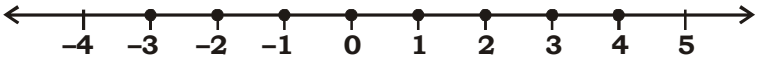
7. p = 8 and q = 4

2009

1. 60 km/h

2. $x = 1.4, 0.39$

3. S.S. = {–3, –2, –1, 0, 1, 2, 3, 4}



4. a = 9 and b = 6

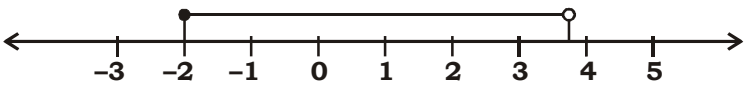
5. 2

6. $x = 2$ and $y = 1$

7. a : b = 3 : 2

2010

2. S.S. = $\left\{x \mid -2 \le x < \frac{15}{4}, x \in \mathrm{R}\right\}$



3. $p = \frac{4}{3}$

4. $p = 1$

5. $\begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$

6. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

8. 20.34

9. $(x - 2)\ (x + 3)\ (2x - 1)$

2011

1. k = 13
   (x + 5) is a factor

2. Yes
   Number of columns of A = Number of rows of B = 2.
   $\begin{bmatrix} 26 \\ 0 \end{bmatrix}$

3. S.S. = {–3, –2, –1, 0, 1}

4. $x = 7$

5. $x = 20$

6. $\begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$

7. 8.2 and –2.2

8. $y = 12,\ x = 3$

2012

1. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. 48 km/h

3. 1.24 and –0.643

5. $-4 \leq x < 5,\ x \in R$

6. (i) 2 × 1

   (ii) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

7. $(x - 2)\ (x + 3)\ (3x - 1)$

8. Ravi = ₹ 200
   Sanjeev = ₹ 280

9. m = –1 or m = 4

2013

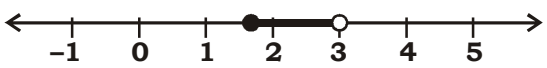
1. $X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$

2. 3

3. a = 5
   b = –11

4. x = 6.53, –1.53

5. $\frac{8}{5} \leq x < 3,\ x \in R$



6. x = 1 and y = 2

7. p = 1 or p = 9

8. $x = \pm 2$

9. ₹ 48

2014

1. $\frac{-9}{4} \leq x < 5, x \in W$

   Solution set = {–2, –1, 0, 1, 2, 3, 4}

2. $x = 3$ and $y = -2$

3. $(x - 1)(x + 13)(x - 2)$

4. $x = 17$

5. (i) $\frac{x}{y} = \frac{5}{3}$

   (ii) $\frac{76}{49}$

6. $\begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$

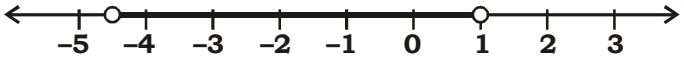
7. $x = 25$

8. $x = 3.6$ or $x = 1.4$

9. 23

2015

1. $-4.5 < x < 1, x \in R$



2. $x = 4$ and $y = -1$

3. k = –4

   other root = – 1

5. a = 3

6. $\begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$

7. 3 and 5

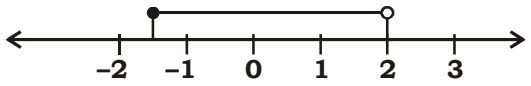
8. $x : y = 2 : 3$

2016

1. k = 13

2. m = –14

3. $\left\{x : \frac{-3}{2} \le x < 2, x \in \mathrm{R}\right\}$



4. $\frac{a}{b} = \frac{4}{3}$

5. $x = 4.9$ or $x = -1.9$

6. (i) The order of matrix X is 2 × 1

   (ii) The matrix X = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

8. 40 km/h

2017

1. Given, b is the mean proportion between a and c.

   $\therefore \quad \frac{a}{b} = \frac{b}{c} = k$ (say)

   $\therefore \quad a = bk, \; b = ck$

   $\therefore \quad a = (ck)k = ck^2, \; b = ck$

   $$\text{L.H.S.} = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

   $$= \frac{(ck^2)^4 + (ck^2)^2(ck)^2 + (ck)^4}{(ck)^4 + (ck)^2c^2 + c^4}$$

   $$= \frac{c^4k^8 + (c^2k^4)(c^2k^2) + c^4k^4}{c^4k^4 + (c^2k^2)c^2 + c^4}$$

   $$= \frac{c^4k^8 + c^4k^6 + c^4k^4}{c^4k^4 + c^4k^2 + c^4}$$

$$= \frac{c^4k^4(k^4+k^2+1)}{c^4(k^4+k^2+1)}$$

$$= k^4$$

$$\text{R.H.S.} = \frac{a^2}{c^2}$$

$$= \frac{(ck^2)^2}{c^2} = \frac{c^2k^4}{c^2}$$

$$= k^4$$

Hence, L.H.S. = R.H.S.

2. $4x^2 - 5x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 4$, $b = -5$ and $c = -3$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25+48}}{8}$$

$$= \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8.544}{8}$$

$$= \frac{13.544}{8} \text{ or } \frac{-3.544}{8}$$

$$= 1.693 \text{ or } -0.443$$

$$= 1.69 \text{ or } -0.44$$

3. **Given :** $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$

Now, $A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\times1+3\times3 & 1\times3+3\times4 \\ 3\times1+4\times3 & 3\times3+4\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times(-2)+1\times(-3) & -2\times1+1\times2 \\ -3\times(-2)+2\times(-3) & -3\times1+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, $A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$$

$5C = A^2 - 5B^2$

$\therefore 5C = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$

$\therefore C = \frac{1}{5}\begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$

$\therefore C = \begin{bmatrix} \frac{5}{5} & \frac{15}{5} \\ \frac{15}{5} & \frac{20}{5} \end{bmatrix}$

$\therefore C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$

4. Let the number to be subtracted from the given polynomial be k.
Let $f(x) = 16x^3 - 8x^2 + 4x + 7 - k$
It is given that $(2x + 1)$ is a factor of $f(x)$.

$\therefore f\left(-\frac{1}{2}\right) = 0$

$\therefore 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$

$\therefore 16 \times \left(-\frac{1}{8}\right) - 8 \times \frac{1}{4} - 2 + 7 - k = 0$

$\therefore -2 - 2 - 2 + 7 - k = 0$

$\therefore 1 - k = 0$

$\therefore \mathbf{k = 1}$

5. $-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2}$, $x \in I$

$\therefore \quad -\frac{17}{2} < -\frac{1}{2} - 4x \le \frac{15}{2}$, $x \in I$

Take $-\frac{17}{2} < -\frac{1}{2} - 4x$	$-\frac{1}{2} - 4x \le \frac{15}{2}$
$-\frac{17}{2} + \frac{1}{2} < -4x$	$-4x \le \frac{15}{2} + \frac{1}{2}$
$-\frac{16}{2} < -4x$	$-4x \le \frac{16}{2}$
$-8 < -4x$	$-4x \le 8$
$2 > x$	$x \ge -2$

$\therefore \quad -2 \le x < 2, \; x \in I$

$\therefore \quad$ S. S. = {–2, –1, 0, 1}

6. **Given :** $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$ and $X = B^2 - 4B$

Now, $B^2 = B \times B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1\times1+1\times8 & 1\times1+1\times3 \\ 8\times1+3\times8 & 8\times1+3\times3 \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix}$

$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$

$X = B^2 - 4B = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Now, $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 5a+0b \\ 0a+5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\therefore \quad 5a = 5 \text{ and } 5b = 50$

$\therefore \quad \boldsymbol{a = 1 \text{ and } b = 10}$

7. (i) $\dfrac{7m+2n}{7m-2n} = \dfrac{5}{3}$

By Componendo – Dividendo, we get

$$\frac{7m+2n+7m-2n}{7m+2n-(7m-2n)} = \frac{5+3}{5-3}$$

$$\therefore \quad \frac{14m}{4n} = \frac{8}{2}$$

$$\therefore \quad \frac{7m}{2n} = \frac{4}{1}$$

$$\therefore \quad \frac{m}{n} = \frac{8}{7}$$

(ii) $\dfrac{m}{n} = \dfrac{8}{7}$

$$\therefore \quad \frac{m^2}{n^2} = \frac{8^2}{7^2}$$

Applying Componendo – Divinendo, we get

$$\therefore \quad \frac{m^2+n^2}{m^2-n^2} = \frac{8^2+7^2}{8^2-7^2}$$

$$\therefore \quad \frac{m^2+n^2}{m^2-n^2} = \frac{64+49}{64-49}$$

$$\therefore \quad \frac{m^2+n^2}{m^2-n^2} = \frac{113}{15}$$

8. Let Vivek's age be x years and Amit's age be $(47 - x)$ years.
According to the given condition,

$x(47 - x) = 550$

$\therefore \quad 47x - x^2 = 550$

$\therefore \quad x^2 - 47x + 550 = 0$

$\therefore \quad x^2 - 25x - 22x + 550 = 0$

$\therefore \quad x(x - 25) - 22(x - 25) = 0$

$\therefore \quad (x - 25)\ (x - 22) = 0$

$\therefore \quad x = 25 \text{ or } x = 22$

So, Vivek's age is 25 years and Amit's age is 22 years.

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